1 IV

$$\begin{cases} u_{xx} = u_{tt} \\ u_x(0,t) = 0 \\ u_x(4,t) = 0 \\ u(x,0) = 2\cos(\pi x) + 5 \\ u_t(x,0) = 1 - \cos(2\pi x) \end{cases}$$

$$u(x,t) = F(x)G(t) \rightarrow F''G = \ddot{G}F \rightarrow \frac{F''}{F} = \frac{\ddot{G}}{G} = \begin{cases} k^2 \\ 0 \\ -k^2 \end{cases}$$

$$\therefore k^2 \rightarrow \frac{F''}{F} = k^2 \rightarrow F'' - k^2 F = 0 \rightarrow F(x) = Ae^{kx} + Be^{-kx}$$

از شرایط مرزی $\rightarrow$ $A = B = 0$

$$u_x(0,t) = F'(0)G(t) = 0 \rightarrow F'(0) = 0$$

$$u_x(4,t) = F'(4)G(t) = 0 \rightarrow F'(4) = 0$$

$$\therefore 0 \rightarrow \frac{F''}{F} = 0 \rightarrow F'' = 0 \rightarrow F(x) = A^*x + B^*$$

$$F'(0) = 0 \rightarrow A^* = 0$$

$$F_0(x) = B^* \rightarrow$$

در مسئله طور صدق نمی‌کند.

$$\ddot{G}(t) = 0 \rightarrow G(t) = (At + B)$$

$$\therefore -k^2 \rightarrow \frac{F''}{F} = -k^2 \rightarrow F'' + k^2 F = 0$$

$$F(x) = A^*\cos kx + B^*\sin kx$$

$$F'(0) = -A^*k\sin kx + B^*k\cos kx\Big|_{x=0} \rightarrow B^* = 0$$

$$F'(4) = 0 \rightarrow -A^*\sin kx\Big|_{x=4} = 0 \xrightarrow{A^* \neq 0} \sin 4k = 0, \quad (1)$$

$$4k = n\pi, \quad k = \frac{n\pi}{4}$$

$$F_n(x) = A^*\cos\left(\frac{n\pi}{4}x\right) \qquad n = 0, 1, 2, \ldots$$

2) $\frac{\ddot{G}}{G} = -\left(\frac{n\pi}{4}\right)^2 \rightarrow \ddot{G} + \left(\frac{n\pi}{4}\right)^2 G = 0 \rightarrow G(t) = \left(C^* \cos\frac{n\pi}{4}t + D^* \sin\frac{n\pi}{4}t\right)$

$$U(x,t) = b_0 t + a_0 + \sum_{n=1}^{\infty} \left(a_n \cos\frac{n\pi}{4}t + b_n \sin\frac{n\pi}{4}t\right)\cos\frac{n\pi}{4}x$$

$$U(x,0) = 2\cos(\pi x) + 5 = a_0 + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi}{4}x$$

$$= a_0 + a_1 \cos\frac{\pi}{4}x + a_2 \cos\frac{\pi}{2}x + a_3 \cos\frac{3\pi}{4}x + a_4 \cos\pi x + \cdots$$

$a_0 = 5$

$a_4 = 2$

$a_1 = a_2 = a_3 = 0$

$a_5 = a_6 = \cdots = 0$

$$U_t(x,0) = b_0 + \sum_{n=1}^{\infty} \left(-a_n \frac{n\pi}{4}\sin\frac{n\pi}{4}t + b_n \frac{n\pi}{4}\cos\frac{n\pi}{4}t\right)\cos\frac{n\pi}{4}x \Big|_{t=0} = 1 - \cos(2\pi x)$$

$$= b_0 + \sum_{n=1}^{\infty} \frac{n\pi}{4} b_n \cos\frac{n\pi}{4}x = 1 - \cos 2\pi x$$

$$= b_0 + \frac{\pi}{4} b_1 \cos\frac{\pi}{4}x + \frac{\pi}{2} b_2 \cos\frac{\pi}{2}x + \frac{3\pi}{4} b_3 \cos\frac{3\pi}{4}x + \cdots + 2\pi b_8 \cos 2\pi x + \cdots$$

$$= 1 - \cos 2\pi x$$

$b_0 = 1$

$b_1 = b_2 = b_3 = \cdots = b_7 = \cdots$

$2\pi b_8 = -1 \rightarrow b_8 = \boxed{\frac{-1}{2\pi}}$

3 B

$$\boxed{u(x,t) = 5 + t + 2\cos\pi x\cos\pi t - \frac{1}{2\pi}\sin 2\pi t\cos 2\pi x}$$

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