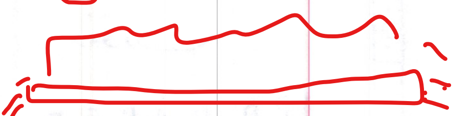
$$\begin{cases} u_t = k u_{xx} \\ u_x(0,t) = 0 \\ u_x(l,t) = 0 \\ u(x,0) = 6 + 4\cos\left(\frac{3\pi}{l}x\right) \end{cases} \longrightarrow \text{جداسازی متغیرها}$$



جداسازی متغیرها:

$$u(x,t) = F(x)G(t) \xrightarrow{\text{مشتق}} F\dot{G} = kF''G \xrightarrow{\div FGk} \frac{F''}{F} = \frac{1}{k}\frac{\dot{G}}{G} = \begin{cases} p^2 \\ 0 \\ -p^2 \end{cases}$$

$\therefore p^2$

$$\frac{F''}{F} = p^2 \qquad F'' - p^2F = 0 \longrightarrow F(x) = Ae^{-px} + Be^{px}$$

$$\begin{cases} \frac{\partial u}{\partial x}(0,t) = 0 \rightarrow F'(0)G(t) = 0 \Rightarrow F'(0) = 0 \longrightarrow A - B = 0 \\ \frac{\partial u}{\partial x}(l,t) = 0 \Rightarrow F'(l)G(t) = 0 \rightarrow F'(l) = 0 \longrightarrow -Ae^{-pl} + Be^{pl} = 0 \end{cases}$$

($G(t) \neq 0$)

$A = B = 0$

$\therefore 0 \longrightarrow \frac{F''}{F} = 0 \longrightarrow F'' = 0, \quad F(x) = A^*x + B^*$

$G = At + B$

$F'(0) = 0 \longrightarrow A^* = 0$

$\frac{1}{k}\frac{\dot{G}}{G} = 0 \rightarrow \dot{G} = 0$

$F'(l) = 0$ صدق می‌کند $\longleftarrow F(x) = B^*$

$\therefore -p^2 \longrightarrow \frac{F''}{F} = -p^2 \rightarrow F'' + p^2F = 0 \longrightarrow F(x) = A^{**}\cos px + B^{**}\sin px$

$$F'(0) = -A^{**}p\sin px + B^{**}p\cos px \Big|_{x=0} = 0 \longrightarrow B^{**} = 0$$

$$F'(l) = -A^{**}p\sin pl = 0 \xrightarrow{A^{**} \neq 0} \sin pl = 0, \quad pl = n\pi, \quad p_n = \frac{n\pi}{l}, \quad n = 1, 2, \ldots$$

$$\boxed{F_n(x) = A_n^{**}\cos\frac{n\pi}{l}x} \qquad n = 1, 2, 3, \ldots$$

$$\frac{1}{K}\frac{\dot{G}}{G} = -\left(\frac{n\pi}{l}\right)^2 \longrightarrow \dot{G} + \left(\frac{n\pi}{l}\right)^2 K G = 0 \quad , \quad \left(\frac{n\pi}{l}\right)^2 K = \lambda_n^2$$

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$$\frac{dG}{dt} = -\lambda_n^2 G \longrightarrow \frac{dG}{G} = -\lambda_n^2 dt$$

$$G_n(t) = c^* e^{-\lambda_n^2 t}$$

$$u(x,t) = a_0 + \sum_{n=1}^{\infty} a_n e^{-\lambda_n^2 t} \cos\frac{n\pi}{l}x$$

$b_0 t$

برای یافتن a_0 و a_n از شرط اولیه کمک می‌گیریم.

$$u(x,0) = 6 + 4\cos\left(\frac{3\pi}{l}x\right) = a_0 + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi}{l}x + b_0 t$$

$$= a_0 + a_1\cos\frac{\pi}{l}x + a_2\cos\frac{2\pi}{l}x + a_3\cos\frac{3\pi}{l}x + \cdots$$

$b_0 t +$

$b_0 = 0$

$a_0 = 6$

$a_3 = 4$

$a_1 = a_2 = 0$

$a_4 = a_5 = a_6 = \cdots = a_n = \cdots = 0$

$$u(x,t) = 6 + 4e^{-\lambda_3^2 t}\cos\frac{3\pi}{l}x = 6 + 4e^{-\frac{9K\pi^2}{l^2}t}\cos\frac{3\pi}{l}x$$

2C