CHAPTER 1

BASIC CONCEPTS

1.1 Convection Heat Transfer

- Examine thermal interaction between a surface and an adjacent moving fluid

1.2 Important Factors in Convection Heat Transfer

- Surface temperature is too high. How to reduce it?

(1) Use a fan
(2) Change the fluid
(3) Increase surface area

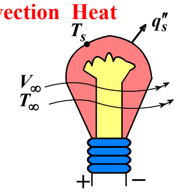


Fig. 1.1

- Conclusion: Three important factors in convection

(1) *fluid motion*
(2) *fluid nature*
(3) *surface geometry*

- Examples of the role of fluid motion in convection:
  - Fanning to feel cool
  - Stirring a mixture of ice and water
  - Blowing on the surface of coffee in a cup
  - Orienting a car radiator to face air flow

1.3 Focal Point in Convection Heat Transfer

Determination of temperature distribution in a moving fluid

$$T = T(x, y, z, t) \qquad (1.1)$$

1.3 Fourier's Law of Conduction

$$q_x \propto \frac{A(T_{si} - T_{so})}{L}$$

$$q_x = k\frac{A(T_{si} - T_{so})}{L} \qquad (1.2)$$

k = *thermal conductivity*

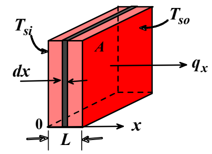


Fig. 1.2

- Valid for:

(1) steady state
(2) constant k
(3) one-dimensional conduction

- **Reformulate to relax restrictions. Consider element *dx***

$$q_x = k\,A\,\frac{T(x)-T(x+dx)}{dx} = -k\,A\,\frac{T(x+dx)-T(x)}{dx}$$

$$q_x = -k\,A\,\frac{dT}{dx} \qquad (1.3)$$

$$q''_x = Heat\ \ flux$$

$$q''_x = \frac{q_x}{A} \qquad (1.4)$$

$$q''_x = -k\frac{dT}{dx} \qquad (1.5)$$

Generalize (1.5):

$$q''_x = -k\frac{\partial T}{\partial x},\quad q''_y = -k\frac{\partial T}{\partial y},\quad q''_z = -k\frac{\partial T}{\partial z} \qquad (1.6)$$

4

(1) Why negative sign?

(2) $k \neq$ constant

(3) Find $T(x,y,z,t)$, use (1.6) to obtain q''

(4) Changing fluid motion changes $T(x,y,z,t)$

1.5 Newton's Law of Cooling

$$q''_s \propto (T_s - T_\infty)$$

q''_s = **surface flux**

T_s = **surface temperature**

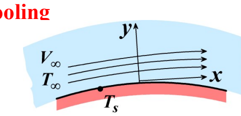


Fig. 1.3

$$q''_s = h(T_s - T_\infty) \qquad (1.7)$$

5

- **Eq. (1.7) is *Newton's law of cooling***
- ***h* is called the *heat transfer co***

$$h = f(\text{geometry, motion, properties}, \Delta T) \qquad (1.8)$$

1.6 The Heat Transfer Coefficient *h*

- **Is *h* a property?**
- **Does *h* depend on temperature distribution?**

Apply Fourier's law

$$q''_s = -k\frac{\partial T(x,0,z)}{\partial y} \qquad (1.9)$$

Combine (1.7) and (1.9)

$$h = -k\frac{\dfrac{\partial T(x,0,z)}{\partial y}}{(T_s - T_\infty)} \qquad (1.10)$$

Temperature distribution in needed to determine *h*

6

- **Apply Newton's law to the bulb:**

$$T_s = T_\infty + \frac{q_s''}{h} \qquad (1.11)$$

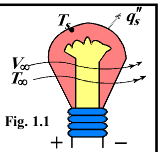


Fig. 1.1

- **Increase V_∞ to increase h and lower T_s**

Table 1.1 Typical values of h

Process	h (W/m^2 − °C)
Free convection	
Gases	5-30
Liquids	20-1000
Forced convection	
Gases	20-300
Liquids	50-20,000
Liquid metals	5,000-50,000
Phase change	
Boiling	2,000-100,000
Condensation	5,000-100,000

1.7 Differential Formulation of Basic Laws

- **Three basic laws:** conservation of mass, momentum, and energy
- **Formulation**
  - ***Differential***
  - ***Integral***
  - ***Finite difference***
- **Key assumption: continuum**

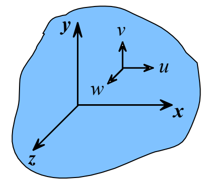


Fig. 1.4

1.8 Mathematical Background

(a) Velocity Vector $\vec{V}$

$$\vec{V} = ui + vj + wk \qquad (1.12)$$

(b) Velocity Derivative

$$\frac{\partial \vec{V}}{\partial x} = \frac{\partial u}{\partial x} i + \frac{\partial v}{\partial x} j + \frac{\partial w}{\partial x} k \qquad (1.13)$$

(c) The Operator ∇

Cartesian: $$\nabla \equiv \frac{\partial}{\partial x} i + \frac{\partial}{\partial x} j + \frac{\partial}{\partial x} k \qquad (1.14)$$

Cylindrical: $$\nabla \equiv \frac{\partial}{\partial r} i_r + \frac{1}{r}\frac{\partial}{\partial \theta} i_\theta + \frac{\partial}{\partial z} i_z \qquad (1.15)$$

Spherical: $$\nabla \equiv \frac{\partial}{\partial r} i_r + \frac{1}{r}\frac{\partial}{\partial \theta} i_\theta + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi} i_\phi \qquad (1.16)$$

(d) Divergence of a Vector

$$div.\vec{V} \equiv \nabla\cdot\vec{V} = \frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z} \qquad (1.17)$$

(e) Derivative of the Divergence

$$\frac{\partial}{\partial x}(\nabla\cdot\vec{V}) = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right) \qquad (1.18)$$

or

$$\frac{\partial}{\partial x}(\nabla\cdot\vec{V}) = \nabla\cdot\frac{\partial}{\partial x}(u\,i+v\,j+w\,k)$$

or

$$\frac{\partial}{\partial x}(\nabla\cdot\vec{V}) = \nabla\cdot\frac{\partial\vec{V}}{\partial x} \qquad (1.19)$$

(f) Gradient of Scalar

$$Grad\ T = \nabla T = \frac{\partial T}{\partial x}i+\frac{\partial T}{\partial y}j+\frac{\partial T}{\partial z}k \qquad (1.22)$$

(g) Total Differential and Total Derivative

f = flow field dependent variable such as u, v, p, etc.

Cartesian Coordinates:

$$f = f(x,y,z,t) \qquad (a)$$

Total differential of f:

$$df = \frac{\partial f}{\partial x}dx+\frac{\partial f}{\partial y}dy+\frac{\partial f}{\partial z}dz+\frac{\partial f}{\partial t}dt$$

or

$$\frac{df}{dt} \equiv \frac{Df}{Dt} = \frac{\partial f}{\partial x}\frac{dx}{dt}+\frac{\partial f}{\partial y}\frac{dy}{dt}+\frac{\partial f}{\partial z}\frac{dz}{dt}+\frac{\partial f}{\partial t} \qquad (b)$$

But

$$\frac{dx}{dt}=u,\quad \frac{dy}{dt}=v,\quad \frac{dz}{dt}=w \qquad (c)$$

Substitute (c) into (b)

$$\frac{df}{dt}=\frac{Df}{Dt}=u\frac{\partial f}{\partial x}+v\frac{\partial f}{\partial y}+w\frac{\partial f}{\partial z}+\frac{\partial f}{\partial t} \qquad (1.21)$$

Total derivative: $\frac{df}{dt}=\frac{Df}{Dt}$

Convective derivative: $u\frac{\partial f}{\partial x}+v\frac{\partial f}{\partial y}+w\frac{\partial f}{\partial z}$ (d)

Local derivative: $\frac{\partial f}{\partial t}$ (e)

Apply to velocity component *u*. Set $f = u$

$$\frac{du}{dt} = \frac{Du}{Dt} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t} \quad (1.22)$$

(1.22) represents

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = \text{convective acceleration in the } x\text{-direction} \quad (f)$$

$$\frac{\partial u}{\partial t} = \text{local acceleration} \quad (g)$$

Cylindrical coordinates : r, θ, z

$$\frac{dv_r}{dt} = \frac{Dv_r}{Dt} = v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z} + \frac{\partial v_r}{\partial t} \quad (1.23a)$$

13

$$\frac{dv_\theta}{dt} = \frac{Dv_\theta}{Dt}$$

$$= v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z\frac{\partial v_\theta}{\partial z} + \frac{\partial v_\theta}{\partial t} \quad (1.23b)$$

$$\frac{dv_z}{dt} = \frac{Dv_z}{Dt} = v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z} + \frac{\partial v_z}{\partial t} \quad (1.23c)$$

- Total derivative of temperature:

set $f = T$ in (1.21)

$$\frac{dT}{dt} = \frac{DT}{Dt} = u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} + \frac{\partial T}{\partial t} \quad (1.24)$$

14

1.9 Problem Solving Format

Solve problems in stages:

(1) Observations
(2) Problem Definition
(3) Solution Plan
(4) Plan Execution
- (i) Assumptions
- (ii) Analysis
- (iii) Computations
- (iv) Checking

(5) Comments

15

1.10 Units

SI units

Length (L): meter (m)
Time (t): second (s)
Mass (m): kilogram (kg)
Temperature (T): kelvin (K)

- *Celsius* and *kelvin* scales

$$T(^{o}C) = T(K) - 273.15 \quad (1.25)$$

Derived units:

- Force: *newton* (N)

One newton = force to accelerate one kilogram one meter per sec per sec:

Force = mass × acceleration

$N = kg . m /s^2$

16

- Energy: *joules* (J)

One joule = energy due to a force of one newton moving a distance of one meter

$J = N. m = kg . m^2 /s^2$

- Power: *watts* (W)

One watt = one joule per second

W = J/s = N. m/s = kg . m2 /s3

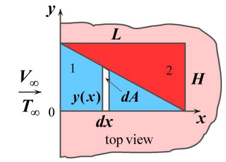
17

Example 1.1: Heat Loss from Identical Triangles

- Surface temperature: T_s
- Variable h:

$$h(x) = \frac{C}{\sqrt{x}}$$

- Determine: $\frac{q_1}{q_2}$



(1) Observations

- Newton's law gives q
- h varies with x
- Integration is required

(2) Problem Definition.

Determine dq for dx of each triangle

18

(3) Solution Plan.

Apply Newton's law to element and integrate

(4) Plan Execution

(i) Assumptions

- steady state
- one-dimensional
- uniform T_∞
- uniform T_s
- negligible radiation

(ii) Analysis

Apply Newton's law

$$dq = h(x)(T_s - T_\infty)dA \quad \text{(a)}$$

$$h = \frac{C}{\sqrt{x}} \quad \text{(b)}$$

Triangle 1: $dA_1 = y_1(x)dx$ (c)

Triangle 2: $dA_2 = y_2(x)dx$ (d)

Geometry: $y_1(x) = \frac{H}{L}(L - x)$ (e)

$$y_2(x) = \frac{H}{L}x \quad \text{(f)}$$

(e) into (c), (f) into (d):

$$dA_1 = \frac{H}{L}(L - x)dx \quad \text{(g)}$$

$$dA_2 = \frac{H}{L}xdx \quad \text{(h)}$$

(b) and (g) into (a), integrate

$$q_1 = \int dq_1 = \int_0^L C(T_s - T_\infty)\frac{H}{L}\frac{L-x}{x^{1/2}}dx$$

$$q_1 = (4/3)C(T_s - T_\infty)HL^{1/2} \quad \text{(i)}$$

Similarly

$$q_2 = \int dq_2 = \int_0^L C(T_s - T_\infty)\frac{H}{L}\frac{x}{x^{1/2}}dx$$

$$q_2 = (2/3)C(T_s - T_\infty)HL^{1/2} \quad \text{(j)}$$

Ratio of (i) and (j)

$$\frac{q_1}{q_2} = 2 \quad \text{(k)}$$

(iii) Checking

Dimensional check:

(b) gives units of C: $C = \text{W/m}^{3/2}\text{-}^{\circ}\text{C}$

(i) gives units of q_1

$$q_1 = C(\mathrm{W/m^{3/2}\text{-}^oC})(T_s - T_\infty)(^o\mathrm{C})H(\mathrm{m})L^{1/2}(\mathrm{m}^{1/2}) = \mathrm{W}$$

Qualitative check:

$q_1 > q_2$ because base of 1 is at x = 0 where $h = \infty$.

(5) Comments

- **Orientation is important.**
  Same area triangles but different q
- **Use same approach for other geometries**