

TABLE OF TRANSFORMATIONS OF REGIONS  
(See Chap. 7)

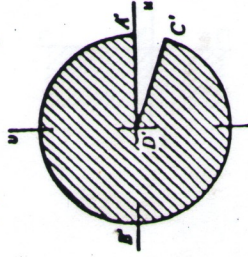
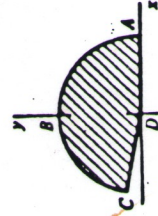


Figure 1.  $w = z^2$ .

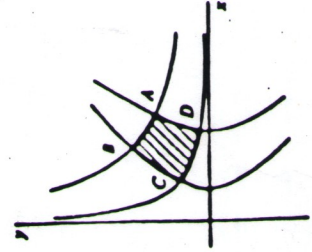
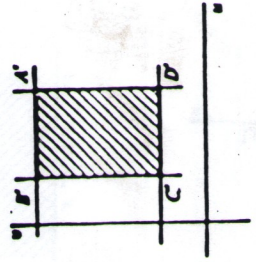


Figure 2.  $w = z^2$ .



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Figure 25.  $w = \frac{h}{a} [(z^2 - 1)^{1/2} + \coth^{-1} z]$ .

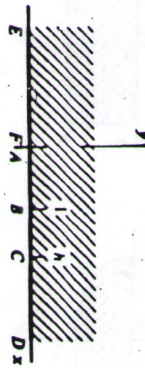
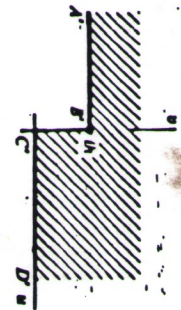
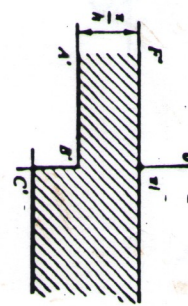


Figure 26.  $w = \coth^{-1} \left( \frac{2z - h - 1}{h - 1} \right) - \frac{1}{h} \coth^{-1} \left[ \frac{(h+1)z - 2h}{(h-1)z} \right]$ .



\*See Exercise 3, Sec. 93.

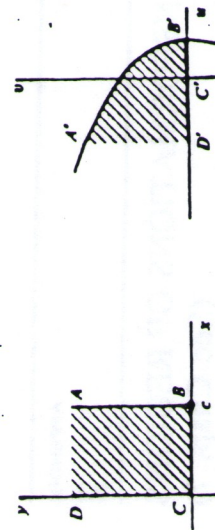


Figure 3.  $w = z^2$ ;  $A'B'$  on parabola  $v^2 = -4c^2(u - c^2)$ .

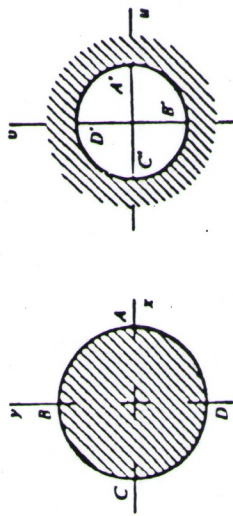


Figure 4.  $w = 1/z$ .

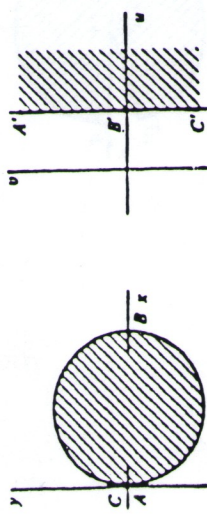


Figure 5.  $w = 1/z$ .

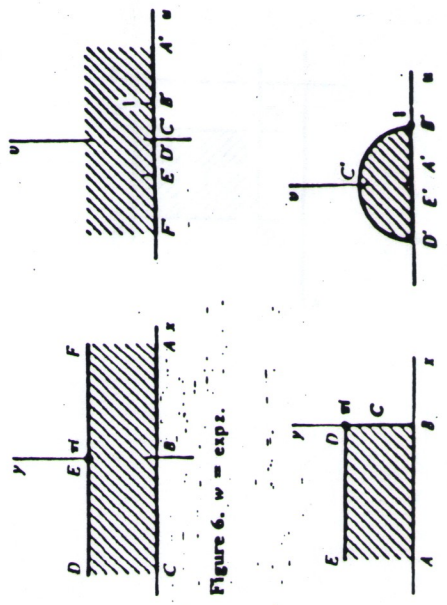


Figure 6.  $w = \exp z$ .

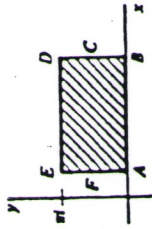


Figure 8.  $w = \exp z$ .

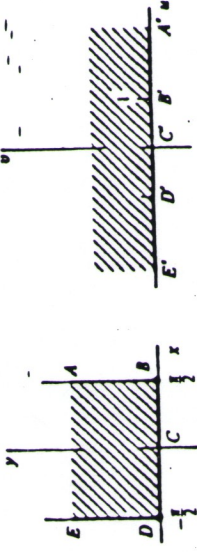


Figure 9.  $w = \sin z$ .

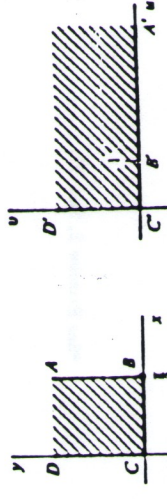


Figure 10.  $w = \sin z$ .

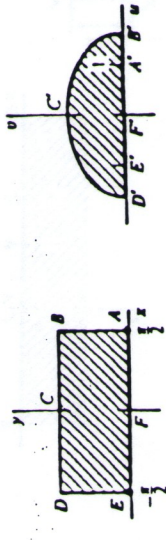


Figure 11.  $w = \sin z$ ;  $BCD$  on line  $y = b$  ( $b > 0$ ),  $B'C'D'$  on ellipse  $\frac{u^2}{\cosh^2 b} + \frac{v^2}{\sinh^2 b} = 1$ .

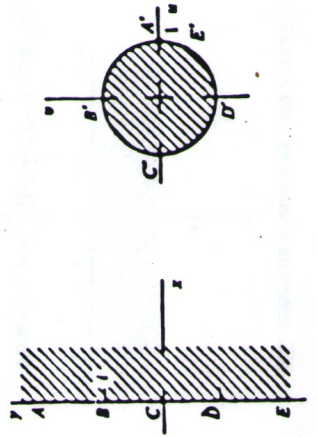


Figure 12.  $w = \frac{z-1}{z+1}$ .

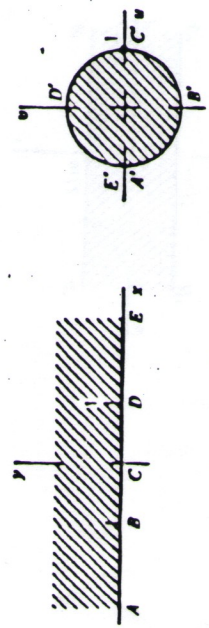


Figure 13.  $w = \frac{z-i}{z+i}$ .

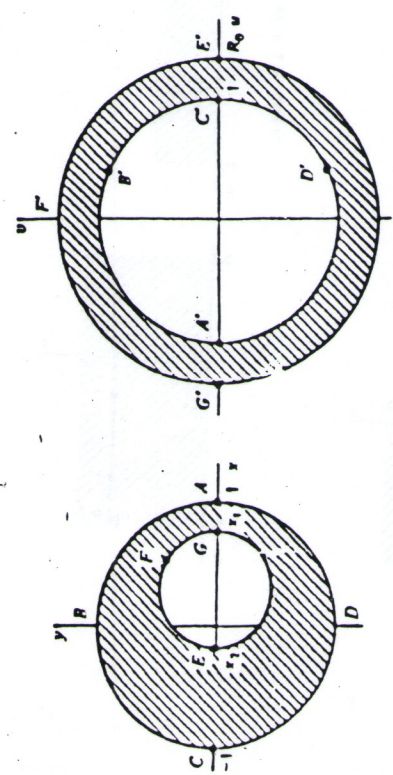


Figure 14.  $w = \frac{z-a}{az-1}$ ;  $a = \frac{1+x_1x_2 + \sqrt{(1-x_1)(1-x_2)}}{x_1+x_2}$   
 $R_0 = \frac{1-x_1x_2 + \sqrt{(1-x_1)(1-x_2)}}{x_1-x_2}$  ( $a > 1$  and  $R_0 > 1$  when  $-1 < x_2 < x_1 < 1$ ).

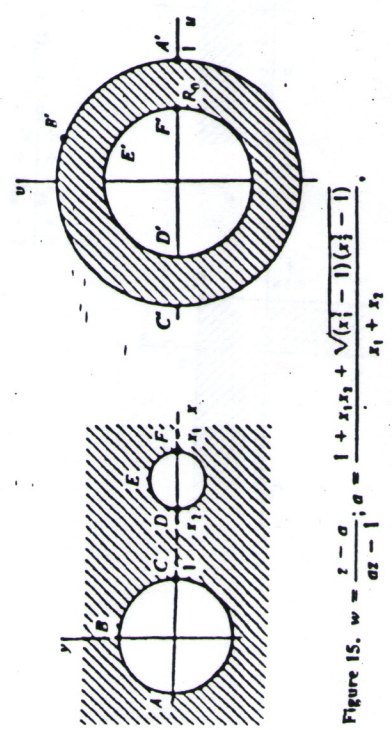


Figure 15.  $w = \frac{z-a}{az-1}$ ;  $a = \frac{1+x_1x_2 + \sqrt{(x_1^2-1)(x_2^2-1)}}{x_1+x_2}$   
 $R_0 = \frac{x_1x_2 - 1 - \sqrt{(x_1^2-1)(x_2^2-1)}}{x_1-x_2}$  ( $x_1 < a < x_2$  and  $0 < R_0 < 1$  when  $1 < x_1 < x_2 < x_1$ ).

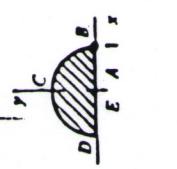


Figure 16.  $w = z + \frac{1}{z}$ .

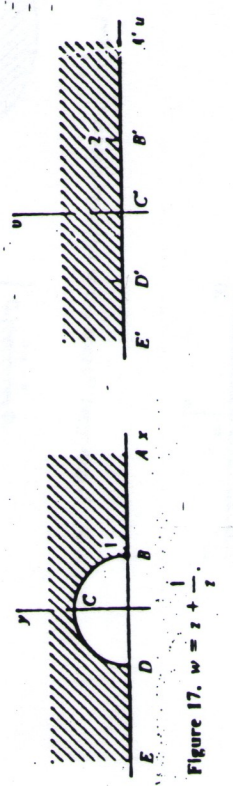


Figure 17.  $w = z + \frac{1}{z}$ .



Figure 18.  $w = z + \frac{1}{z}$ ;  $B'C'D'$  on ellipse  $\frac{u^2}{(b+1/b)^2} + \frac{v^2}{(b-1/b)^2} = 1$ .

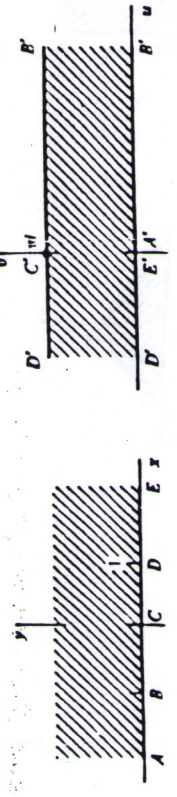


Figure 19.  $w = \text{Log} \frac{z-1}{z+1}$ ;  $z = -\coth \frac{w}{2}$ .

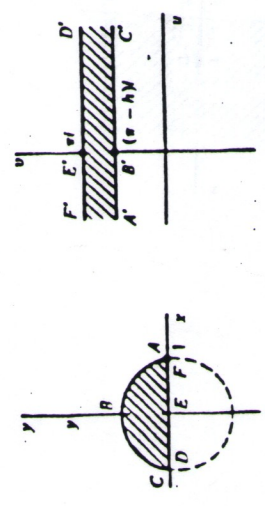


Figure 20.  $w = \text{Log} \frac{z-1}{z+1}$ ;  $ABC$  on circle  $x^2 + (y + \cot h)^2 = \csc^2 h$  ( $0 < h < \pi$ ).

ART. 2

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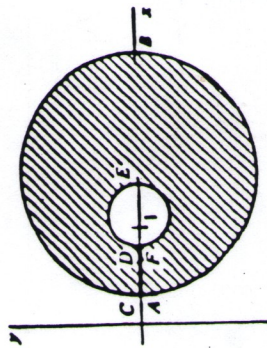


Figure 21.  $w = \text{Log} \frac{z+1}{z-1}$ ; centers of circles at  $z = \coth c_1$ , radii: each  $c_1$ , ( $n = 1, 2$ ).

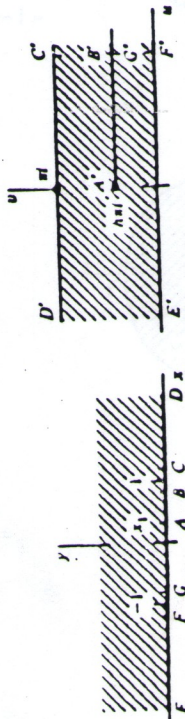


Figure 22.  $w = h \ln \frac{z+1}{z-1} + \ln 2(1-h) + i\pi - h \text{Log}(z+1) - (1-h) \text{Log}(z-1)$ ;

$x_1 = 2h - 1.$

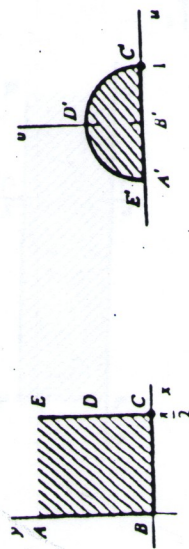


Figure 23.  $w = \left(\tan \frac{z}{2}\right)^2 = \frac{1 - \cos z}{1 + \cos z}$ .

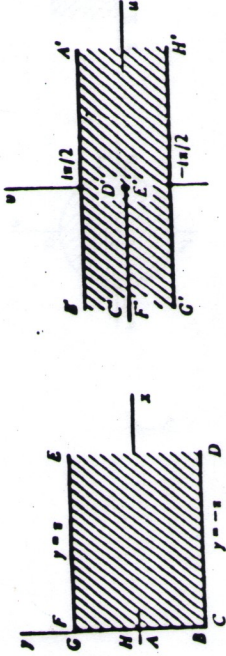


Figure 25.  $w = \text{Log} \left( \coth \frac{z}{2} \right)$ .

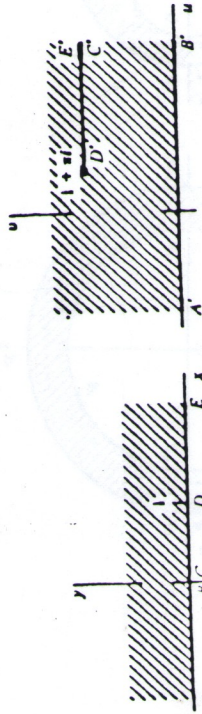


Figure 26.  $w = \pi i + z - \text{Log} z.$

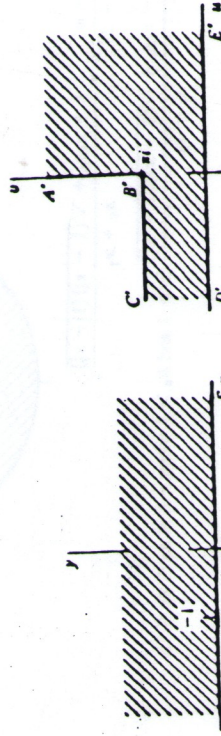


Figure 27.  $w = 2(z+1)^{1/2} + \text{Log} \frac{(z+1)^{1/2} - 1}{(z+1)^{1/2} + 1}$ .

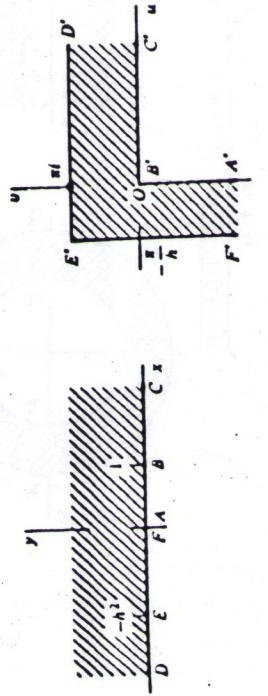


Figure 28.  $w = -i \ln \frac{1 + i \coth z + 1}{1 + i \coth z - 1} = \left( \frac{z-1}{z+1} \right)^{1/2}$ .